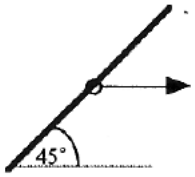
MECHANICS (C) UNIT 1 — TEST PAPER 8

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A golf ball and a table tennis ball are dropped together from the top of a building. The golf ball hits the ground after 1·7 seconds.
   Calculate the height of the top of the building above the ground. [3]
   According to a simple model, the two balls hit the ground at the same time.
   State why this may not be true in practice and describe a refinement to the model which could lead to a more realistic solution. [2]

2. A particle is in equilibrium under the action of three forces **P**, **Q** and **R** acting in the same horizontal plane. **P** has magnitude 9 N and acts on a bearing of 030°. **Q** has magnitude 12 N. and acts on a bearing of 225°. Find the magnitude and direction of **R**. [7]

3. A small ring, of mass m kg, can slide along a straight wire which is fixed at an angle of 45° to the horizontal as shown. The coefficient of friction between the ring and the wire is $\frac{2}{7}$.



   The ring rests in equilibrium on the wire and is just prevented from sliding down the wire when a horizontal string is attached to it, as shown
   Show that the tension in the string has magnitude $\frac{5mg}{9}$ N. [7]

4. The velocity, v ms^{-1}, of a particle at time t s is given by $v = 4t^2 - 9$.
   (i) Find the acceleration of the particle when it is instantaneously at rest. [3]
   (ii) Find the distance travelled by the particle from time $t = 0$ until it comes to rest. [4]

5. Two model cars A and B have masses 200 g and k g respectively. They move towards each other in a straight line and collide directly when their speeds are 5 ms^{-1} and 4 ms^{-1} respectively. As a result the speed of A is reduced to 2 ms^{-1}, in the same direction as before. The direction of B's motion is reversed and its speed immediately after the impact is 5 ms^{-1}.
   (i) Find the value of k. [3]
   The surface on which the cars move is rough, and B comes to rest 3 seconds after the impact. The coefficient of friction between both cars and the surface is μ.
   (ii) Find the value of μ. [3]
   (iii) Find the distance travelled by A after the impact before it comes to rest. [3]

6. Two cyclists, Alice and Bobbie, travel from P to Q along a straight path. Alice starts from rest at P just as Bobbie passes her at 3·5 ms^{-1}. Bobbie continues at this speed while Alice accelerates at 0·2-ms^{-2} for T seconds until she attains her maximum speed. At this moment both cyclists immediately start to slow down, with constant but different decelerations, and they come to rest at Q 80 seconds after Alice started moving.

   (i) Sketch, on the same diagram, the velocity-time graphs for the two cyclists. [4]

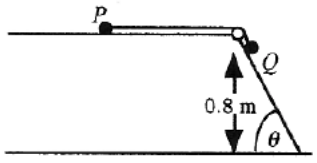
   By using the fact that both cyclists cover the same distance, find

   (ii) the value of T, [4]

   (iii) the distance between P and Q, [2]

   (iv) the magnitude of Bobbie's deceleration. [2]

7. Two particles P and Q, of masses $2m$ and $3m$ respectively, are connected by a light string. Initially, P is at rest on a smooth horizontal table. The string passes over a small smooth pulley and Q rests on a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.



   The coefficient of friction between Q and the inclined plane is $\frac{1}{6}$.

   The system is released from rest with Q at a distance of 0·8 metres above a horizontal floor.

   (i) Show that the acceleration of P and Q is $\frac{21g}{50}$, stating a modelling assumption which you must make to ensure that both particles have the same acceleration. [7]

   (ii) Find the speed with which Q hits the floor. [2]

   After Q hits the floor and does not rebound, P moves a further 0·2 m until it hits the pulley.

   (iii) Find the total time after the system is released before P hits the pulley. [4]

MECHANICS 1 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

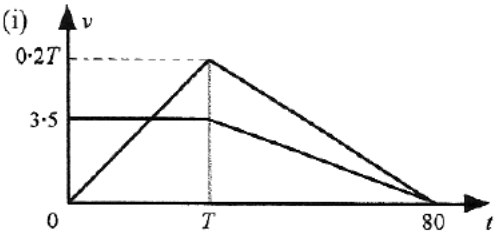
1. $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 9{\cdot}8 \times 1{\cdot}7^2 = 14{\cdot}2$ m — M1 A1 A1
   Lighter ball may be more affected by air resistance : include this — B1 B1 — 5

2. Total force to north $= 9\cos 30° - 12\cos 45° = -0{\cdot}691$ N — M1 A1
   Total force to east $= 9\sin 30° - 12\sin 45° = -3{\cdot}985$ N — M1 A1
   $|\mathbf{R}| = \sqrt{(3{\cdot}985^2 + 0{\cdot}691^2)} = 4{\cdot}04$ N, on bearing $\tan^{-1} 5{\cdot}77 = 080{\cdot}1°$ — M1 A1 A1 — 7

3. Resolve horizontally : $T + \frac{2}{7}\frac{R}{\sqrt{2}} = \frac{R}{\sqrt{2}}$ $\quad T = \frac{5R}{7\sqrt{2}}$ — M1 A1
   Resolve vertically : $mg = \frac{R}{\sqrt{2}} + \frac{2}{7}\frac{R}{\sqrt{2}} = \frac{9R}{7\sqrt{2}}$ $\quad R = \frac{7\sqrt{2}}{9}mg$ — M1 A1 A1
   $T = \frac{5}{7\sqrt{2}} \times \frac{7\sqrt{2}}{9}mg = \frac{5mg}{9}$ — M1 A1 — 7

4. (i) When $v = 0$, $4t^2 = 9$ $\quad t = 1{\cdot}5$ $\quad a = 8t = 12$ ms^{-2} — M1 A1 A1
   (ii) $s = \int_0^{1{\cdot}5} v\,dt = [\frac{4}{3}t^3 - 9t]_0^{1{\cdot}5} = 4{\cdot}5 - 13{\cdot}5$, so distance $= 9$ m — M1 M1 A1 A1 — 7

5. (i) $200 \times 5 - 4k = 200 \times 2 + 5k$ $\quad 9k = 600$ $\quad k = 66\frac{2}{3}$ — M1 A1 A1
   (ii) $v = u + at$: $0 = 5 + 3a$ $\quad a = -\frac{5}{3}$ $\quad \mu g = \frac{5}{3}$ $\quad \mu = 0{\cdot}170$ — M1 A1 A1
   (iii) $v^2 = u^2 + 2as$: $0 = 4 + 2(-\frac{5}{3})s$ $\quad s = 1{\cdot}2$ m — M1 A1 A1 — 9

6. (i)


   B2 B2
   (ii) Areas under graphs equal : $40(0{\cdot}2T) = 1{\cdot}75(T + 80)$ — M1 A1 A1
   $6{\cdot}25T = 140$ $\quad T = 22{\cdot}4$ — A1
   (iii) Area $= 8T$, so distance $= 179{\cdot}2$ m — M1 A1
   (iv) $3{\cdot}5 \div (80 - T) = 0{\cdot}0608$ ms^{-2} — M1 A1 — 12

7. (i) Modelling assumption : string is inextensible — B1
   $F = ma$: $T = 2ma$, $\quad 3mg\sin\theta - \frac{1}{6}(3mg\cos\theta) - T = 3ma$ — M1 A1 A1
   Add : $3mg(0{\cdot}8) - 0{\cdot}5mg(0{\cdot}6) = 5ma$ $\quad 5a = 2{\cdot}1g$ $\quad a = \frac{21g}{50}$ — M1 A1 A1
   (ii) Dist. $= 1$ m : $v^2 = 2(\frac{21g}{50})(1)$ $\quad v = 2{\cdot}87$ ms^{-1} — M1 A1
   (iii) Time for Q to reach floor is t where $1 = 0{\cdot}21gt^2$ $\quad t = 0{\cdot}697$ s — M1 A1
   0·2 m at 2·87 ms^{-1} takes 0·0697 s, so total time $= 0{\cdot}767$ s — M1 A1 — 13